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MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate session number

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Thursday 3 May 2012 (afternoon)

Examination code

2	2	1	2	–	7	2	0	3

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

14 pages

Please **do not** write on this page.

Answers written on this page will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

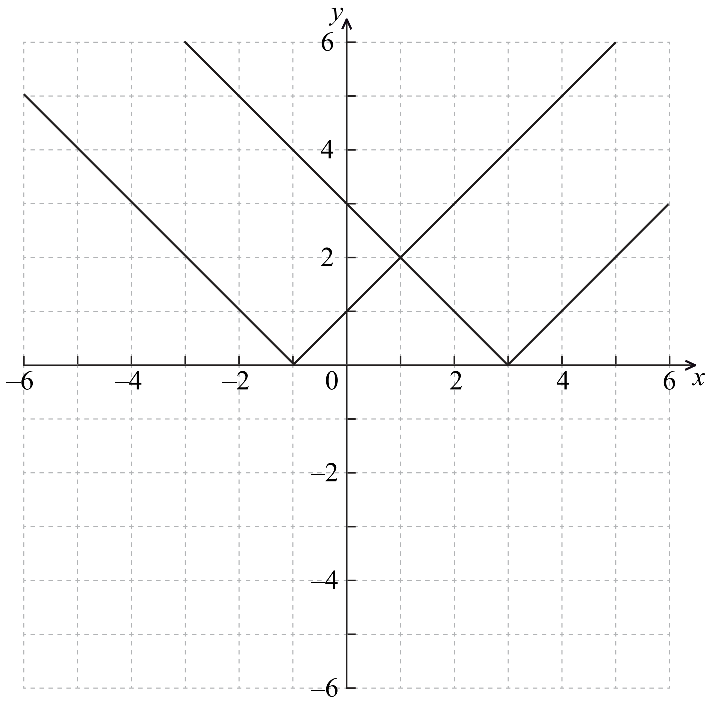
*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

Find the value of k if $\sum_{r=1}^{\infty} k\left(\frac{1}{3}\right)^{r} = 7$.

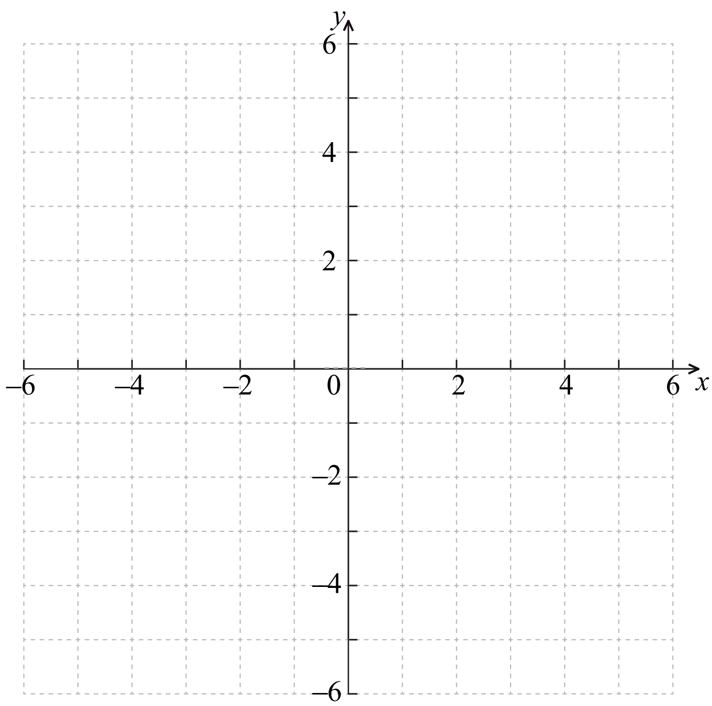
2. *[Maximum mark: 8]*

The graphs of $y = |x+1|$ and $y = |x-3|$ are shown below.



Let $f(x) = |x+1| - |x-3|$.

(a) Draw the graph of $y = f(x)$ on the blank grid below. *[4 marks]*



(This question continues on the following page)

(Question 2 continued)

(b) Hence state the value of

(i) $f'(-3)$;

(ii) $f'(2.7)$;

(iii) $\int_{-3}^{-2} f(x)\,\mathrm{d}x$. *[4 marks]*

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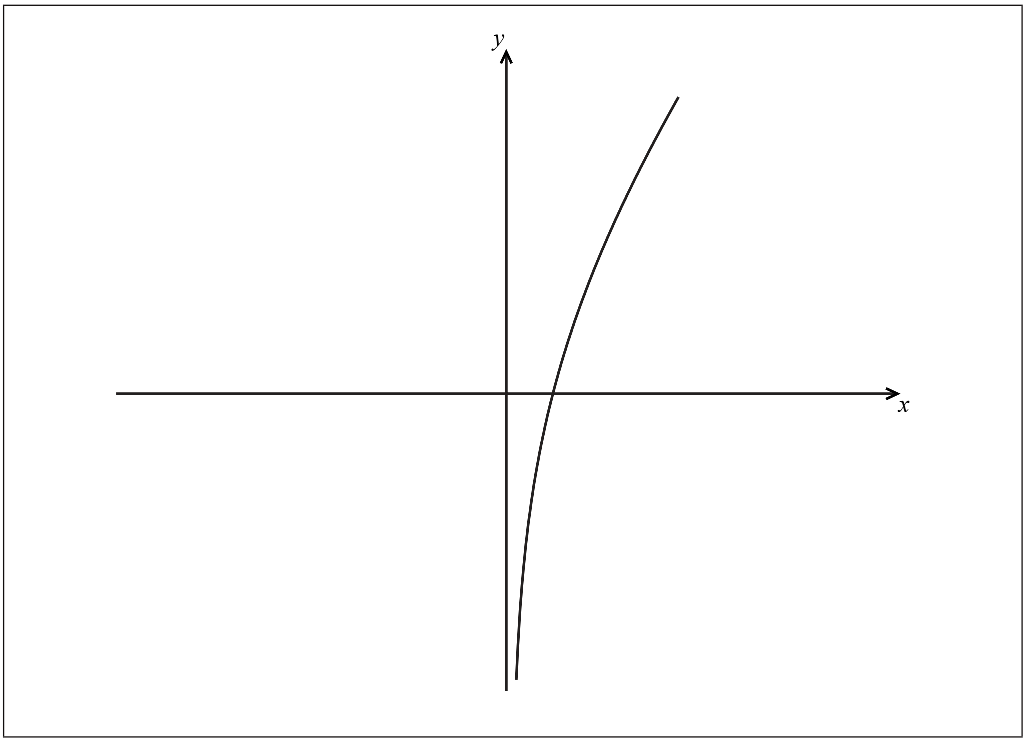
3. *[Maximum mark: 7]*

If $z_1 = a + a\sqrt{3}\,\mathrm{i}$ and $z_2 = 1 - \mathrm{i}$, where a is a real constant, express z_1 and z_2 in the form $r\,\mathrm{cis}\,\theta$, and hence find an expression for $\left(\frac{z_1}{z_2}\right)^6$ in terms of a and i.

4. *[Maximum mark: 6]*

The graph below shows $y = f(x)$, where $f(x) = x + \ln x$.

(a) On the graph below, sketch the curve $y = f^{-1}(x)$. *[2 marks]*



(b) Find the coordinates of the point of intersection of the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. *[4 marks]*

5. *[Maximum mark: 7]*

Let $f(x) = \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$.

(a) For what values of x does $f(x)$ not exist? *[2 marks]*

(b) Simplify the expression $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$. *[5 marks]*

6. *[Maximum mark: 8]*

The graph below shows the two curves $y=\frac{1}{x}$ and $y=\frac{k}{x}$, where $k>1$.

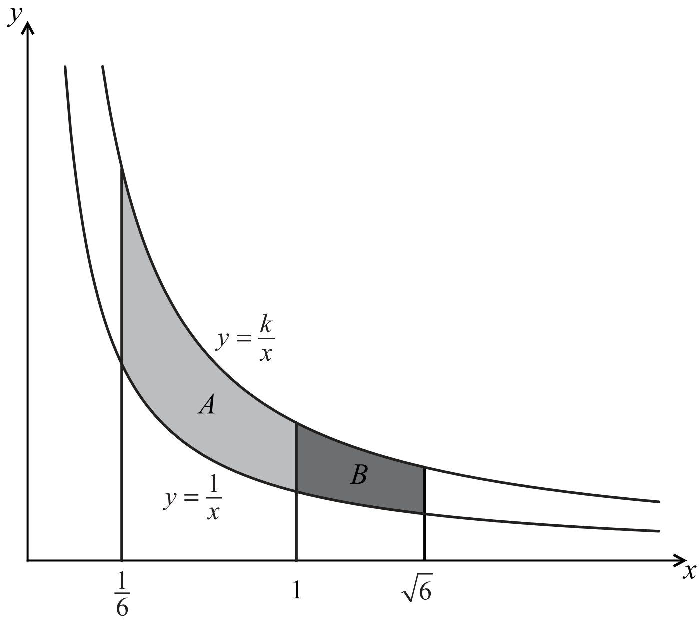


diagram not to scale

(a) Find the area of region A in terms of k. *[3 marks]*

(b) Find the area of region B in terms of k. *[2 marks]*

(c) Find the ratio of the area of region A to the area of region B. *[3 marks]*

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7. *[Maximum mark: 6]*

Given that z is the complex number $x+\mathrm{i}y$ and that $|z|+z=6-2\mathrm{i}$, find the value of x and the value of y.

8. *[Maximum mark: 5]*

Solve the equation $2 - \log_3(x+7) = \log_{\frac{1}{3}} 2x$.

9. *[Maximum mark: 9]*

The curve C has equation $2x^2 + y^2 = 18$. Determine the coordinates of the four points on C at which the normal passes through the point $(1, 0)$.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

10. *[Maximum mark: 21]*

In the triangle ABC, $A\hat{B}C = 90^\circ$, $AC = \sqrt{2}$ and $AB = BC + 1$.

(a) Show that $\cos \hat{A} - \sin \hat{A} = \frac{1}{\sqrt{2}}$. *[3 marks]*

(b) By squaring both sides of the equation in part (a), solve the equation to find the angles in the triangle. *[8 marks]*

(c) Apply Pythagoras' theorem in the triangle ABC to find BC, and hence show that $\sin \hat{A} = \frac{\sqrt{6} - \sqrt{2}}{4}$. *[6 marks]*

(d) Hence, or otherwise, calculate the length of the perpendicular from B to [AC]. *[4 marks]*

11. *[Maximum mark: 17]*

(a) $\boldsymbol{A}$ and $\boldsymbol{U}$ are square matrices, and $\boldsymbol{X} = \boldsymbol{U}^{-1}\boldsymbol{A}\boldsymbol{U}$. Use mathematical induction to prove that $\boldsymbol{X}^n = \boldsymbol{U}^{-1}\boldsymbol{A}^n\boldsymbol{U}$, for $n \in \mathbb{Z}^+$. *[7 marks]*

(b) Let $\boldsymbol{A} = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}$ and $\boldsymbol{U} = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$.

(i) Find the matrix $\boldsymbol{D}$ such that $\boldsymbol{AU} = \boldsymbol{UD}$.

(ii) Write down the matrix $\boldsymbol{D}^2$.

(iii) **Hence** prove that $\boldsymbol{A}^{2n} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, for $n \in \mathbb{Z}^+$.

(iv) Using the result from part (iii), show that $(\boldsymbol{A}^n)^{-1} = \boldsymbol{A}^n$, for $n \in \mathbb{Z}^+$. *[10 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 22]*

Let $f(x)=\sqrt{\frac{x}{1-x}}$, $0<x<1$.

(a) Show that $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}(1-x)^{-\frac{3}{2}}$ and deduce that f is an increasing function. *[5 marks]*

(b) Show that the curve $y=f(x)$ has one point of inflexion, and find its coordinates. *[6 marks]*

(c) Use the substitution $x=\sin^2\theta$ to show that $\int f(x)\,\mathrm{d}x=\arcsin\sqrt{x}-\sqrt{x-x^2}+c$. *[11 marks]*

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